Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

(2) $4a^2x^2 + 4abx + b^2 = 4ac + b^2$. Extracting square root of (2) we get

(3) $$2ax + b = \sqrt{(4ac + b^2)}.$$

Now equation (3) can always be obtained directly from (1) without using (2) by observing the law of its formation, viz.; that the first member of (3) is the first derived polynomial of (1). Therefore all complete quadratic equations can be solved by the following rule:

1. Reduce to the form $ax^2 + bx = c$.
2. Make the first derived polynomial $= \pm\sqrt{(b^2 + 4ac)}$, using the double sign, and the roots of this equation are those required.

Examles. — 1. $3x^2 - 27x = -42$, to find the values of x.

The first derived polynomial is $6x - 27$; $\therefore 6x - 27 = \sqrt{(729 - 504)} = \pm 15$; $\therefore x = 7$ or 2.

2. $3\sqrt[3]{x^2} - 10\sqrt[3]{x} = -3$, to find the values of x.

Put $\sqrt[3]{x} = y$; then $\sqrt[3]{x^2} = y^2$. Therefore the given equation becomes $3y^2 - 10y = -3$. First derived polynomial is $6y - 10$, $\therefore 6y - 10 = \sqrt{(100 - 36)} = \pm 8$; $\therefore y = 3$ or $\frac{1}{3}$; $\therefore x = 27$ or $\frac{1}{27}$.

3. $\frac{1}{3}x^2 + 5x = 42$, to find the value of x.

The first derived polynomial is $\frac{2}{3}x + 5$; $\therefore \frac{2}{3}x + 5 = \sqrt{(25 + 56)} = \pm 9$; $\therefore x = 6$ or -21.

It will be seen that if the first derived polynomial is made equal to 0, the resulting value of x is that required for a maximum or a minimum.

The works of Davies, Robinson, Ray and Schuyler do not contain this method of solving quadratics. I discovered it last October; but it is probable that the method has long been known, though it may never have been published.

SOLUTION OF A PROBLEM.

BY THEO. L. DE LAND, WASHINGTON, D. C.

Supposing the sky, all the way from the zenith to the horizon, to be thickly dotted with stars, and that they are equally distributed in every part, what would be the mean of all their altitudes?

Solution. — Let n be the number of stars, h the altitude of any one of them, and A its azimuth.

The area of an elementary portion of the sky is $\cos h.dhdA$ which must be taken as constant.

$$\text{Mean altitude} = \frac{h_1 + h_2 + h_3 + \ldots + h_n}{n}.$$

Multiplying both numerator and denominator by the above factor, bearing in mind that the denominator then becomes the whole sky, we have

$$\text{Mean altitude} = \frac{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} h \cos h\, dh dA}{2\pi}$$

$$= \int_0^{\frac{\pi}{2}} h \cos h dh \qquad = \Big[h \sin h + \cos h\Big]_0^{\frac{\pi}{2}}$$

$$= \tfrac{1}{2}\pi - 1.$$

$\frac{1}{2}\pi$ in angular measure is 90°; and 1 is an are equal to the radius.

$\therefore$ Mean altitude = 90° — 57° 17′ 44.8″ = 32° 42′ 15.2″.

SOLUTON OF A PROBLEM.

BY PROF. C. M. WOODWARD, ST. LOUIS, MO.

To find in terms of the coordinates of a point (x',y') on one branch of the equilateral hyperbola $xy = m$, the length of a normal dropped from (x',y') upon the other branch.

Let the equation to the normal be

$$\frac{x - x'}{c} = \frac{y - y'}{s} = l$$

or $\qquad x = cl + x' \qquad$ and $y = sl + y'$.

Multiplying we have $\quad xy = l^2sc + l(x's + y'c) + x'y'$, but $xy = m$, and $x'y' = m$ for the intersections, hence $l^2sc + l(x's + y'c) = 0$.

$$l = \frac{x's + y'c}{sc} = \frac{x'}{c} + \frac{y'}{s} = x' \sec\theta + y' \operatorname{cosec}\theta.$$

(neglecting the negative sign, as that has reference only to the direction of measurement).

Now this distance must be a minimum; so put

$$\frac{dl}{d\theta} = x'\tan\theta\sec\theta - y'\cot\theta\cos\theta = 0.$$

$$\tan^3\theta = \frac{y'}{x'} \qquad \tan\theta = \frac{\sqrt[3]{y'}}{\sqrt[3]{x'}},$$

$$\sec\theta = \frac{\sqrt{(\sqrt[3]{x'^2} + \sqrt[3]{y'^2})}}{\sqrt[3]{x'}} \qquad \operatorname{cosec}\theta = \frac{\sqrt{(\sqrt[3]{x'^2} + \sqrt[3]{y'^2})}}{\sqrt[3]{y'}},$$

$$\therefore l = x'^{\frac{2}{3}}\left(x'^{\frac{2}{3}} + y'^{\frac{2}{3}}\right)^{\frac{1}{2}} + y'^{\frac{2}{3}}\left(x'^{\frac{2}{3}} + y'^{\frac{2}{3}}\right)^{\frac{1}{2}} = \left(x'^{\frac{2}{3}} + y'^{\frac{2}{3}}\right)^{\frac{3}{2}}.$$